Lecture 3: The Nuclear Landscape and Excited States: Notes

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

January 16, 2026

1 The Chart of the Nuclides: The Nuclear Map (Lamarsh §2.8)

To navigate nuclear engineering, we must move beyond the 1D Periodic Table (ordered by Z) to a 2D map: the **Chart of the Nuclides**. Here, we plot the number of protons (Z) on the vertical axis and the number of neutrons (N) on the horizontal axis. Each square represents a specific isotope.

1.1 The Valley of Stability

If we were to plot the "energy" or mass of every possible combination of N and Z, we would see a "Valley of Stability."

- **Light Nuclei ($A < 40$):** Stability follows the $N = Z$ line (e.g., ^{12}C, ^{16}O). At low mass, the Strong Force easily overcomes the minimal Coulomb repulsion of a few protons.
- **Heavy Nuclei:** The line of stability curves toward the neutron axis. To overcome the cumulative **Coulomb Repulsion** of many protons, the nucleus requires extra "nuclear glue" in the form of a higher proportion of neutrons ($N/Z > 1$).

1.2 Magic Numbers and Shell Structure

Similar to noble gases in chemistry, certain "Magic Numbers" of nucleons (2, 8, 20, 28, 50, 82, 126) create exceptionally stable nuclei.

- These numbers correspond to filled quantum shells within the nucleus.
- **Engineering Significance:** Nuclei with magic numbers of neutrons are more stable and often have very low "neutron absorption cross-sections" (though the low absorption arises from detailed nuclear structure rather than 'magic' alone), making them useful as structural materials in reactors (e.g., Zirconium, $N = 50$).
- **Doubly Magic:** $^{208}_{82}$Pb (82 protons, 126 neutrons) is the heaviest stable isotope. Being "doubly magic" makes it a very deep part of the stability valley.

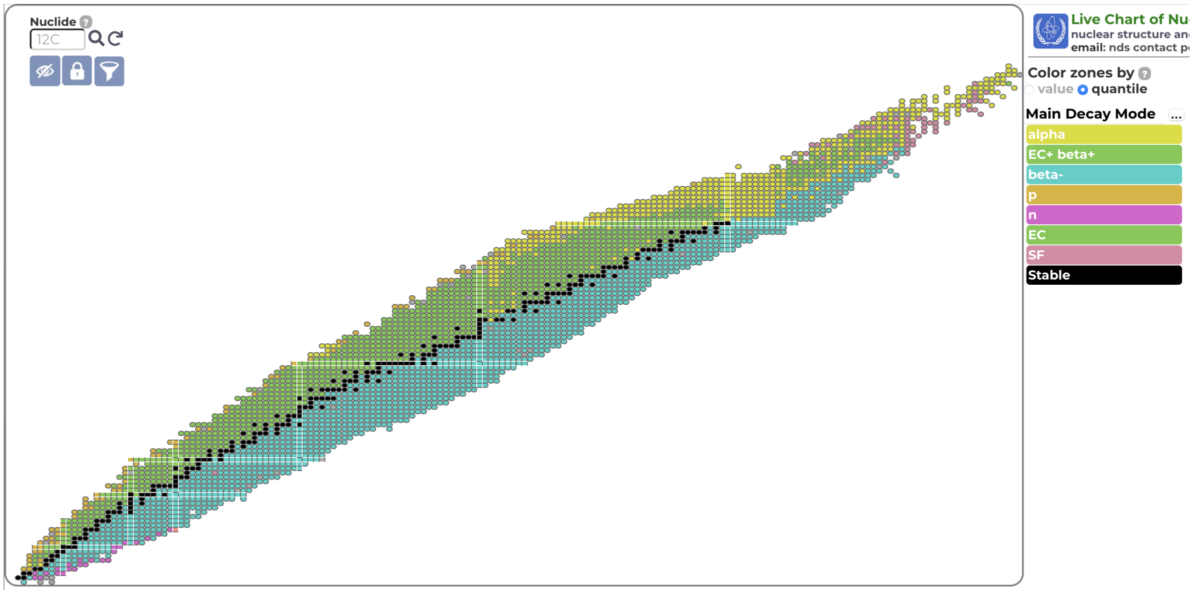

Figure 1: Chart of the Nuclides. Source: IAEA Live Chart of Nuclides.
Note: Vertical axis = Z, Horizontal axis = N (neutron number). The highlighted vertical and horizontal strips correspond to "magic numbers" of protons and neutrons.

2 The Binding Energy Curve (Lamarsh §2.11)

The "altitude" of an isotope on our map is determined by its **Binding Energy** (BE). As established in Lecture 2, mass is a property of energy. When nucleons bind together, they release energy, resulting in a **Mass Defect** (Δm):

$$\Delta m = [Z \cdot M(^1H) + N \cdot m_n] - M(^A_Z X) \tag{1}$$

2.1 Example: The "Altitude" of the Deuteron (^{2}H)

The Deuteron is the simplest bound system (1 proton, 1 neutron). Using the neutral atom convention:

1. **Mass of Parts:** $M(^1H) + m_n = 1.007825 + 1.008665 = 2.016490$ u.
2. **Measured Mass of Whole:** $M(^2H) = 2.014102$ u.
3. **Mass Defect:** $\Delta m = 2.016490 - 2.014102 = 0.002388$ u.
4. **Total BE:** 0.002388 u $\times$ 931.5 MeV/u = **2.224** MeV.

2.2 Binding Energy per Nucleon (BE/A)

To compare the stability of different elements, we look at $E_b = BE/A$. This represents the average energy required to "chip off" a single nucleon from the nucleus.

- **The Iron Peak (^{56}Fe):** At ∼8.8 MeV/nucleon, Iron-56 is very nearly the most tightly bound nucleus. It is the "bottom" of the energy valley.

- **Fusion:** Light elements (like H) move "up the curve" (releasing energy) by combining.
- **Fission:** Heavy elements (like U) move "up the curve" (releasing energy) by splitting.

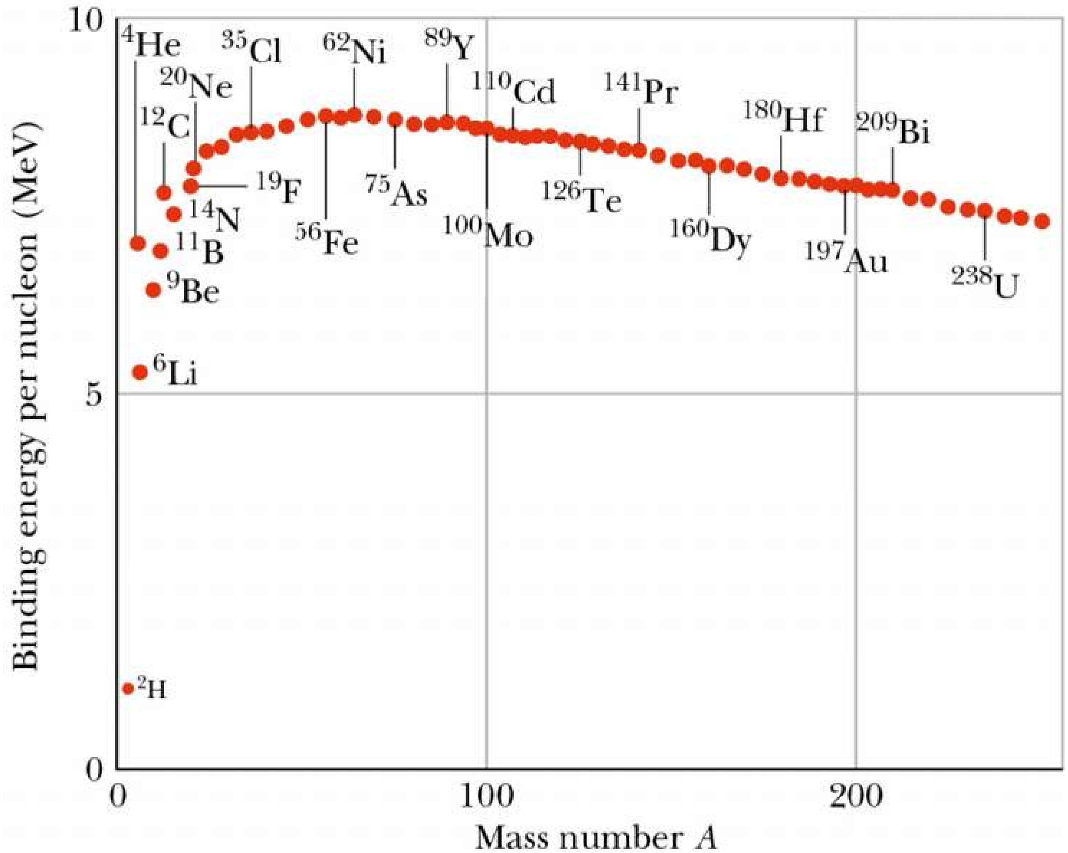

Figure 2: The curve of Binding Energy per Nucleon (BE/A) as a function of Mass Number (A). Note the rapid rise for light nuclei (fusion region) and the gradual decline for heavy nuclei (fission region) after the peak at ^{56}Fe. Source: physics.stackexchange.com.

> *The Star Connection:* Stars spend their lives building elements up the BE/A curve through fusion. Once they hit Iron, fusion no longer releases energy—it consumes it. This is why stars "die" once they produce iron cores, leading to the supernovae and neutron stars we discussed previously.

3 Excited States and Gamma Emission (Lamarsh §2.7)

Just as electrons in an atom occupy discrete energy shells, the protons and neutrons within a nucleus occupy specific quantum states governed by the Nuclear Shell Model. When a nucleus possesses energy in excess of its most stable configuration, it is in an **excited state**.

3.1 Nuclear Energy Levels

The spacing between nuclear energy levels is significantly larger than those in the electron cloud.

- **Ground State:** The lowest energy configuration of a nucleus.
- **Excited State (X^*):** A configuration where one or more nucleons have been promoted to higher energy shells.
- **Transition Energy:** When a nucleus transitions from an excited state E_2 to a lower state E_1, it must shed energy equal to $\Delta E = E_2 - E_1$.

3.2 Gamma Radiation (γ)

Unlike an excited electron which may emit visible or UV light ($\sim$eV range), a nucleus "relaxes" by emitting a high-energy photon called a **Gamma Ray**.

- **Energy Scale:** Typically in the range of 10 keV to several MeV.
- **Nature of Emission:** Because gamma rays are photons (electromagnetic radiation), the process of gamma decay does not change the number of protons or neutrons (Z and A remain constant).
- **Conservation:** The energy of the emitted photon is slightly less than the transition energy due to the "recoil" of the nucleus (the momentum associated with a gamma ray photon is conserved), though in engineering practice, we often assume $E_\gamma \approx \Delta E$.

3.3 Nuclear Isomers and Metastability

Typically, excited states are extremely short-lived, decaying in less than 10^{-12} seconds. However, certain states are "forbidden" from decaying quickly due to large differences in nuclear spin between the excited state and the ground state.

- **Isomer:** A long-lived excited state of a nucleus.
- **Metastable State:** If an isomer has a measurable half-life (typically $> 10^{-9}$ seconds), it is designated with the letter **"m"** (e.g., ^{135m}Ba).
- **Isomeric Transition (IT):** The process by which a metastable state eventually decays to the ground state via gamma emission.

  Engineering Context: In reactor physics, metastable states can act as "energy reservoirs." Delayed gamma emission from fission products can contribute to the "decay heat" that must be managed even after a reactor is shut down.

4 Case Study: Technetium-99m in Nuclear Medicine

The physics of metastable states finds its most profound practical application in clinical diagnosis. Technetium-99m (^{99m}Tc) is the "workhorse" of nuclear medicine, accounting for over 80% of all diagnostic scans worldwide.

4.1 The Isomeric Transition of ^{99m}Tc

The "m" in ^{99m}Tc signifies a metastable state. It decays to its ground state (^{99}Tc) via **Isomeric Transition (IT)**, releasing a monoenergetic gamma ray:

$$^{99m}_{43}\text{Tc} \longrightarrow {}^{99}_{43}\text{Tc} + \gamma + \text{recoil} \tag{2}$$

- **Photon Energy:** $E_\gamma = 140.5$ keV.
- **Half-life ($T_{1/2}$):** 6.01 hours.
- **Recoil:** To conserve momentum ($p = E/c$), the nucleus must recoil in the opposite direction of the photon. While critical for the physics, the recoil energy is negligible ($\sim 10^{-2}$ eV) compared to the 140 keV photon.

4.2 Medical Engineering: The Coronary Stress Test

In a myocardial perfusion scan (stress test), ^{99m}Tc is complexed with a pharmaceutical (like *sestamibi*) that is taken up by healthy heart muscle.

- **The Procedure:** The tracer is injected during peak physical stress. A **Gamma Camera** (scintillation detector) rotates around the patient to capture the 140 keV "flashes."
- **Diagnosis:** If an area of the heart appears as a "cold spot" (low photon counts), it indicates restricted blood flow—often a precursor to a myocardial infarction.

4.3 Optimization: Why ^{99m}Tc?

^{99m}Tc is a masterpiece of engineering optimization:

1. **Ideal Energy:** 140 keV is high enough to exit the body without significant attenuation, but low enough to be captured efficiently by lead-collimated cameras.
2. **Patient Safety:** It is a "clean" emitter. It lacks the highly ionizing alpha or beta particles that would deliver a high radiation dose to healthy tissue.
3. **Logistics (The "Moly Cow"):** Because a 6-hour half-life is too short for global shipping, hospitals use a **^{99}Mo/^{99m}Tc Generator**.
    - **Parent Decay:** Hospitals purchase the parent ^{99}Mo ($T_{1/2} = 66$ hrs). It undergoes **Beta (β^-) decay**, where a neutron in the Molybdenum nucleus transforms into a proton:

    $$^{99}_{42}\text{Mo} \longrightarrow {}^{99m}_{43}\text{Tc} + e^- + \bar{\nu}_e$$

    Approximately 87% of these decays "land" in the metastable state (^{99m}Tc) rather than the ground state.
    - **Elution:** The ^{99m}Tc is "milked" or eluted daily using a saline wash. Because the Technetium is chemically different from the Molybdenum, the saline wash selectively strips the Technetium off the generator column, providing a fresh supply of the tracer on-site.

Teacher's Note: The engineering challenge here is a "just-in-time" supply chain. If the nuclear reactors producing the parent ^{99}Mo (like the NRU reactor in Canada or HFR in the Netherlands) go offline, global cardiac diagnostics can grind to a halt within days.

4.4 Recommended Resources and References

1. **Clinical Overview:** Technetium-99m in Myocardial Perfusion Imaging – A summary of how lipophilic tracers lodge in mitochondrial-rich heart cells to map blood flow.

2. **Medical Physics:** StatPearls (NIH): Technetium-99m – A robust reference on the pharmacokinetics of ^{99m}Tc and why its 140 keV energy is considered the "gold standard" for imaging.

3. **Instrumentation:** The Gamma Camera: Basic Principles (U. Michigan) – A technical perspective on the Anger Logic used to determine coordinates of gamma hits.

4. **IAEA Physics Handbook:** Nuclear Medicine Physics Handbook – The authoritative international reference for the formal derivation of detector efficiency and spatial resolution.

5. **Clinical Protocols:** UW Madison Myocardial Stress/Rest Protocol – A real-world hospital protocol showing timing, dosages, and the "stress vs. rest" sequence.

5 Exit Hook: The "Lonely Neutron"

To wrap up our first week, we move from the complex, multi-nucleon stability of the ^{99m}Tc nucleus to the simplest, most vulnerable component of nuclear matter: the free neutron.

5.1 The Stability Paradox

Within the "potential well" of a stable nucleus, a neutron can exist indefinitely. However, once removed from the collective, the neutron is unstable.

- **The Decay:** A free neutron is "unhappy." Within a mean lifetime of about $\tau \approx 880s$ (half-life $T_{1/2} = \tau\, ln(2)$), it will undergo **Beta Decay** to become a proton:

$$n \longrightarrow p + e^- + \bar{\nu}_e + 0.78 \text{ MeV} \tag{3}$$

- **Mass-Energy Balance:** This decay is energetically possible because the mass of a neutron is slightly greater than the combined mass of a proton and an electron. That "missing mass" is released as 0.78 MeV of kinetic energy.

5.2 Engineering Utility: Probing the Invisible

If neutrons are so unstable, why do we care about them as individual particles? Because they have no charge, they can "ghost" through the electron clouds of atoms to interact directly with the nuclei.

- **SANS (Small-Angle Neutron Scattering):** We use beams of these "lonely" neutrons to image things X-rays cannot. Because neutrons are sensitive to light elements (like Hydrogen) and possess a magnetic moment, they are the preferred tool for studying the structure of polymers, biological membranes, and the internal stresses in structural materials.

  Preview for Week 2: On Wednesday, we will dive deep into the math of the "Lonely Neutron." We will calculate the exact mass-energy balance of its decay and explore how we harness neutron beams to trigger the fission reactions that power the world.

Lecture 3 Addendum: Atomic vs. Nuclear Binding Energy

1 1. The Atomic Scale: Hydrogen Binding Energy

Note: We must use high-precision CODATA 2022 values. Standard 4-digit precision would show zero mass defect here.

A. The Mass Defect Calculation

- Mass of Proton (m_p): 1.007 276 466 u
- Mass of Electron (m_e): 0.000 548 579 u
- **Sum of Constituents: 1.007 825 045** u
- Mass of Neutral Hydrogen (^{1}H): **1.007 825 032** u

The Mass Defect (Δm) is the difference:

$$\Delta m = (m_p + m_e) - m_{^1\text{H}}$$
$$\Delta m = 1.007\,825\,045 - 1.007\,825\,032$$
$$\Delta m = \mathbf{0.000\,000\,013}\text{ u} \quad (1.3 \times 10^{-8}\text{ u})$$

B. Energy Equivalent (Ionization Energy)

We convert this tiny mass into energy.

$$1\text{ u} \approx 931.494\text{ MeV} \approx 931.5 \times 10^6\text{ eV}$$

$$E_b = (1.3 \times 10^{-8}\text{ u}) \times (931.5 \times 10^6\text{ eV/u})$$
$$E_b \approx \mathbf{13.6}\text{ eV}$$

This matches the experimentally observed ionization energy of Hydrogen.

C. The "Formation" Photon

If a free proton and electron recombine, this energy is released as a photon (UV light). Using $E = 13.606$ eV (precise limit):

- **Frequency (ν):**

$$\nu = \frac{E}{h} = \frac{13.606 \times 1.602 \times 10^{-19}\text{ J}}{6.626 \times 10^{-34}\text{ J s}} \approx \mathbf{3.29 \times 10^{15}}\text{ Hz}$$

- **Wavelength (λ):**

$$\lambda = \frac{c}{\nu} = \frac{2.998 \times 10^8\text{ m/s}}{3.29 \times 10^{15}\text{ Hz}} \approx 9.11 \times 10^{-8}\text{ m} = \mathbf{91.1}\text{ nm}$$

(This is the Lyman-alpha limit, in the Ultraviolet).

---

2 2. The Nuclear Scale: Neutron Decay

Now we look at the Neutron (n). Is it just a proton and electron stuck together?

A. Mass Comparison

- Mass of Neutron (m_n): **1.008 664 916** u
- Mass of Proton + Electron: **1.007 825 045** u

B. Stability Analysis

The Neutron is **heavier** than the sum of a proton and electron.

$$\Delta m = 1.008\,664\,916 - 1.007\,825\,045 = \mathbf{0.000\,839\,871} \text{ u}$$

Since mass is energy ($E = mc^2$), the neutron is like a battery containing excess energy.

- **Reaction:** $p + e \to n$ is **Endothermic**. (You must add energy to force them together).
- **Reaction:** $n \to p + e$ is **Exothermic**. (The neutron spontaneously decays).

C. Energy Release (Q-Value)

How much energy is released when a neutron dies?

$$Q = 0.000\,839\,871 \text{ u} \times 931.494 \text{ MeV/u} \approx \mathbf{0.782} \text{ MeV}$$

D. Where does the energy go?

In the decay $n \longrightarrow p + e^- + \bar{\nu}_e$:

- **Kinetic Energy:** The 0.782 MeV is shared between the proton, electron, and antineutrino as kinetic energy.
- **The Gamma Ray?** In *free* neutron decay, there is typically **no gamma ray**. Gamma rays are emitted when a nucleus is left in an excited state and relaxes. The resulting proton is a simple particle with no excited internal states to relax from, so the energy release is purely kinetic.

Neutron Capture Properties of Zirconium Isotopes

Nuclear Data Summary | January 24, 2026

Overview

The following table details the thermal neutron capture cross-sections (σ_γ) for Zirconium isotopes. Note the distinction between the "Magic Number" isotope (^{90}Zr) and the odd-neutron isotope (^{91}Zr).

Isotopes ^{93}Zr and ^{95}Zr are unstable fission products; their natural abundance is effectively zero, but they possess significant capture cross-sections relevant to reactor physics and burnup calculations.

Isotopic Data

Table 1: Thermal neutron capture cross-sections and abundances for Zirconium isotopes.

Isotope	**Abundance (%)**	**Cross Section** (σ_γ **[b]**)	**Half-Life / Stability**
^{90}Zr	51.45	0.011	Stable ($N = 50$)
^{91}Zr	11.22	1.240	Stable
^{92}Zr	17.15	0.220	Stable
^{93}Zr	0.00	1.300	1.61×10^6 y
^{94}Zr	17.38	0.050	Stable
^{95}Zr	0.00	1.200	64.03 d
^{96}Zr	2.80	0.023	Stable

Weighted Average Cross Section

The effective capture cross-section for natural Zirconium is calculated by summing the abundance-weighted contributions of the stable isotopes:

$$\sigma_{\text{nat}} = \sum_i f_i \sigma_i$$

Using the values from the table above:

$$\sigma_{\text{nat}} \approx (0.5145 \times 0.011) + (0.1122 \times 1.240) + (0.1715 \times 0.220) + (0.1738 \times 0.050) + (0.0280 \times 0.023)$$

$$\sigma_{\text{nat}} \approx 0.0057 + \mathbf{0.1391} + 0.0377 + 0.0087 + 0.0006$$

$$\sigma_{\text{nat}} \approx \mathbf{0.192} \text{ barns}$$

Analysis: Despite having an abundance of only $\approx 11\%$, **^{91}Zr dominates the total absorption**, contributing over **72%** of the total neutron capture cross-section (0.139/0.192). This is due to the high pairing energy required by the unpaired neutron in ^{91}Zr, contrasting sharply with the "magic" stability of ^{90}Zr.